

Syllabus

Course Description

Course Title	Optimisation
Course Code	42169
Course Title Additional	
Scientific-Disciplinary Sector	MATH-06/A
Language	English
Degree Course	Bachelor in Industrial and Mechanical Engineering
Other Degree Courses (Loaned)	
Lecturers	Dr. Saman Babaiekafaki, Saman.Babaiekafaki@unibz.it https://www.unibz.it/en/faculties/engineering/academic-staff/person/48578
Teaching Assistant	
Semester	Second semester
Course Year/s	2
CP	6
Teaching Hours	40
Lab Hours	20
Individual Study Hours	90
Planned Office Hours	18
Contents Summary	<p>The course mainly aims to acquaint students with practical continuous nonlinear optimization models and algorithms, as well as the optimization with MATLAB. At the end of the course, the students are expected to be able to formulate a real-world optimization problem in the framework of a nonlinear programming model, analyze various optimality features of the model, suggest suitable algorithms for solving the model, and finally, determine an approximation of the optimal solution of the model using MATLAB (or another software).</p> <ul style="list-style-type: none"> ¿ Practical Optimization Models ¿ Least Squares Models

	<ul style="list-style-type: none"> ζ First Order Algorithms ζ Second Order Algorithms ζ Convexity and Convex Optimization ζ The KKT Conditions and Duality Theory ζ Topics in Data Mining and Regression Analysis
Course Topics	<ul style="list-style-type: none"> - Nonlinear Optimization Modelling: Formulaic Structure of Nonlinear Optimization Models, Fundamental Models in Production Planning, Support Vector Machine, Energy Capacity Planning, Portfolio/Inventory Optimization, Facility Location, Engineering Design (Geometric Optimization), Regression, Optimal Control, and Robotics Motion Planning - Mathematical Preliminaries and Topological Aspects of Nonlinear Optimization: Vector/Matrix Norms and Nonlinear Multivariable Function Approximation - Optimality Conditions for Unconstrained Optimization Models: Formulaic Structure of Unconstrained Optimization Models, First/Second Order Analysis of Optimality, and Necessary and Sufficient Optimality Conditions - Least Squares Models: Data Fitting, Noise Cancellation, and Circle Fitting - First Order Algorithms for Unconstrained Optimization: Line Search, Gradient Method with Convergence Analysis, Gauss–Newton Method for Nonlinear Least Squares Models, and a Gradient-Based Fixed-Point Iteration Method for Solving the Fermat–Weber Problem - Second Order Algorithms for Unconstrained Optimization: Newton’s Method with Convergence Analysis - Convex Sets: Definition, Convex Balls in Various Norms, Algebraic Operations Preserving Convexity, Convex Hulls, Convex Cones, Conic Hulls, and Topological Properties of Convex Sets - Convex Functions: Definition, Jensen’s Inequality, First/Second Order Analysis of Convexity, Global Optimality, Well-Known Convex Functions, Operations Preserving Functional Convexity, Level Sets and Epigraphs, and Quasi-Convex Functions - Convex Optimization: Formulaic Structure, Global Optimality, Convex Quadratic Models, Chebyshev Center of a Set of Points, Analysis of the Markowitz Portfolio Optimization Model, Orthogonal Projection Methods, Analysis of Linear Classification Models, and Convex Form of the Trust Region Subproblem - Optimization Over Convex Sets: Stationarity and Optimality

	<p>Conditions, Gradient Projection Method with Convergence Analysis, Sparsity Constrained Optimization Models, and Iterative Hard-Thresholding Method</p> <ul style="list-style-type: none"> - Linearly Constrained Nonlinear Optimization Models: Formulaic Structure, Karush–Kuhn–Tucker (KKT) Conditions, Lagrangian Function, Orthogonal Projection onto Half-Spaces, and Orthogonal Regression - KKT Conditions for Equality/Inequality Constrained Nonlinear Optimization Models: Feasible Descent Directions, Fritz–John Conditions, KKT Conditions, Sufficiency of KKT Conditions for Convex Optimization, Analysis of Constrained Least Squares Models, Second Order Optimality Conditions, and Total Least Squares Models - Duality Theory in Nonlinear Optimization: Dual Model Definition, Weak and Strong Duality, Duality Gap and Optimality Bounds, Dual Models of Well-Known Nonlinear Optimization Problems, and Regularization and Denoising - Complementary Topics: Penalty and Barrier Methods for Nonlinear Optimization, and Alternating Direction Method of Multipliers (ADMM) Algorithms
Keywords	<p>Nonlinear programming, Unconstrained optimization, Constrained optimization, Modelling, Applied optimization, Large scale optimization, Least squares, Data fitting, Convex optimization, Optimality conditions, Algorithmic optimization, Line search, Global convergence, Local convergence, CVX software</p>
Recommended Prerequisites	<p>The students should be familiar with the basic concepts of linear algebra and calculus.</p>
Propaedeutic Courses	
Teaching Format	<p>Lectures: The course is delivered using lecture slides, which are regularly uploaded to Microsoft Teams. Additional explanations and derivations are provided on the board to clarify concepts and complement the slide material.</p> <p>Exercises: Topic-oriented exercises are solved, primarily on the board, during class to reinforce students' understanding and help them internalize the theoretical concepts.</p> <p>Software Laboratory: To illustrate the practical implementation and performance of optimization algorithms, main algorithms are</p>

	implemented and tested in the MATLAB environment during dedicated laboratory sessions.
Mandatory Attendance	Highly recommended (not compulsory).
Specific Educational Objectives and Learning Outcomes	<p>Intended Learning Outcomes (ILO)</p> <p>Knowledge and Understanding:</p> <ol style="list-style-type: none"> 1. Knowledge of the main concepts of the nonlinear optimization theory 2. Understanding of the analytical origins of the optimization algorithms 3. Knowledge of the optimization applications in data mining and machine learning <p>Applying Knowledge and Understanding:</p> <ol style="list-style-type: none"> 4. Ability to formulate some real-world problems in the framework of the nonlinear optimization models 5. Ability to deal with some problems in the fields of data mining and machine learning <p>Making Judgments:</p> <ol style="list-style-type: none"> 6. Ability to evaluate reliability of the nonlinear optimization models 7. Ability to assess efficiency of the nonlinear optimization algorithms <p>Communication Skills:</p> <ol style="list-style-type: none"> 8. Ability to interpret different parts of the classic optimization models 9. Ability to analyse performance of the nonlinear optimization algorithms based on the computational results 10. Ability to conduct post-optimal analysis <p>Learning Skills:</p> <ol style="list-style-type: none"> 11. Ability to modify classic nonlinear optimization models for specific real-world problems 12. Capability to adapt classic nonlinear optimization algorithms for high-dimensional optimization models 13. Ability to design (use) software to solve the practical optimization models.
Specific Educational Objectives and Learning Outcomes (additional info.)	
Assessment	- Formative Assessments: Implemented through midterm exams, in-class activities and discussions, which together reinforce

	<p>students' understanding of the course material.</p> <ul style="list-style-type: none"> - Summative Assessments: Students' knowledge is additionally assessed through a final examination, which includes: <ul style="list-style-type: none"> - A written exam; - An oral exam (Optional); - A course project (Optional). <p>The detailed structure of the assessment is presented as follows:</p> <ul style="list-style-type: none"> - 40% Formative Assessments; ILOs assessed: 1 - 12; - 40% Final Exam: Computation; Duration: 2 hours or more; ILOs assessed: 5, 6, 7, 9, 10; - 20% Final Exam: Theory; Duration: 1 hours or less; ILOs assessed: 1, 4; - Oral Exam (Optional); ILOs assessed: 2, 8.; - Course Project (Optional); ILOs assessed: 3, 11, 12, 13.
Evaluation Criteria	<ul style="list-style-type: none"> - Formative Assessments: At least one and at most two midterm exams are held during the semester. In addition, students' participation and engagement in in-class exercises and activities are taken into account as part of the formative assessment. - Final (Written) Exam: The main part of the final exam is devoted to numerical problems in which students are required to implement algorithmic approaches for selected problems. In addition, the exam includes theoretical questions that require students to analyze the convergence behavior of algorithms, discuss specific aspects of the mathematical models, and evaluate the accuracy of the solutions. - Oral Exam: Students may choose to participate in an oral examination, during which their understanding of the fundamental concepts covered in the course is assessed. - Course Project: Software implementation and programming activities constitute an integral part of the course project. In addition, students are encouraged to formulate and solve a real-world optimization problem in order to strengthen their practical experience in modeling and solving optimization problems. <p>* Note: Students who, for any reason, are unable to participate in part or all of the formative assessment may compensate for the</p>

	missing assessment components by completing them as part of the final written examination.
Required Readings	<p>- Amir Beck, <i>Introduction to Nonlinear Optimization: Theory, Algorithms, and Applications with MATLAB</i>, 2nd Edition, SIAM: Philadelphia, 2023.</p> <p>https://sites.google.com/site/amirbeck314/books</p>
Supplementary Readings	<p>- Amir Beck, <i>First Order Methods in Optimization</i>, SIAM: Philadelphia, 2017.</p> <p>- Jorge Nocedal and Stephen J. Wright, <i>Numerical Optimization</i>, Springer: New York, 2006.</p>
Further Information	Software: MATLAB
Sustainable Development Goals (SDGs)	Quality education, Industry, innovation and infrastructure, Decent work and economic growth, Affordable and clean energy